DIAKOPTICS AND THE MULTILEVEL MOMENTS METHOD FOR PLANAR CIRCUITS

Steven Ooms (N.F.W.O.) and Daniël De Zutter
Department of Information Technology (INTEC), University of Gent
St.-Pietersnieuwstraat 41, B-9000 Gent, Belgium
Tel: 32-9-264.3316 Fax: 32-9-264.3593
E-mail: steven.ooms@intec.rug.ac.be.

Abstract—**The Method of Moments matrix equation, resulting from the electromagnetic simulation of a high frequency planar circuit can take much memory and time to solve, typically $\mathcal{O}(n^2)$ resp. $\mathcal{O}(n^3)$. This paper proposes a new solution strategy combining a Multilevel Moments Method scheme with a Modified Diakoptics technique and a Block Gauss-Seidel iterative technique, effectively reducing the solution time.**

I. INTRODUCTION

Electromagnetic simulations based on the Method of Moments [1] are highly accurate and applicable for the design of microwave planar circuits [2]. However, the resulting matrix equation can take much memory and time to solve, typically $\mathcal{O}(n^2)$ resp. $\mathcal{O}(n^3)$. In the past several techniques have been proposed for rendering the solution proces more efficient: multilevel techniques [3], diakoptics [4] and iterative techniques. This paper combines several of these techniques in an innovative way. First the circuit is divided into s subcircuits, which are solved separately using a MOM technique. The resulting current profiles are then combined and refined based on Modified Diakoptics [5] using the efficient Block Gauss-Seidel iterative technique [6]. After convergence these diakoptic profiles are used in a second-level MOM simulation. It can be proven that the results obtained with this technique match the exact results after convergence.

As we work with block matrices, the largest amount of memory needed for this technique is the amount needed for the largest block matrix involved: $\mathcal{O}((n/s)^2)$ with s being the number of divisions. During the (linear) iteration phase these matrices do not change. A numerical example will show that even for a small number of unknowns ($n = 825$) this technique reduces the solution time with a factor of 4 for an accuracy of -80 dB on the S-parameters.

II. MULTILEVEL MOMENTS METHOD

In our approach we apply a two-level Multilevel Moments Method scheme with rooftop basis functions on the lowest level to planar microwave circuits. We will illustrate the theoretical framework of our approach using a microstrip transmission line (Fig. 1a) with two exterior ports P_1 and P_2. First the circuit is divided into S subcircuits (here $S = 4$) by inserting the artificial ports AP_1, AP_2 and AP_3 (Fig. 1b). Each subcircuit is then simulated separately using a Method of Moments by exciting the lowerlevel ports $\mathrm{LP_k}$; the eight current density profiles resulting from these lowerlevel MOM's are shown in Fig. 1c. They can be thought of as "generalized half rooftop" functions at the lowerlevel ports. By demanding current continuity at the portsides of the artificial ports $\mathrm{AP_l}$, these profiles are combined into five "generalized (full) rooftop" functions $\mathrm{BF_k}$ over the lowerlevel ports $\mathrm{LP_k}$ (Fig. 1d). These are used as a set of basis functions for the upperlevel MOM.

The amplitudes A_{ke} of these basis functions $\mathrm{BF_k}$ under the excitation of exterior port $\mathrm{P_e}$ are found with the upperlevel MOM. This results in the system:

$$\sum_{l \in \mathrm{LP}} Z^{\mathrm{up}}_{kl} A_{le} = V^{\mathrm{up}}_{ke}$$

LP is the set of the lowerlevel ports and V^{up}_{ke} the excitation term due to the exterior port $\mathrm{P_e}$. The upperlevel Z-matrix elements Z^{up}_{kl} describe the coupling between two basis functions $\mathrm{BF_k}$ and $\mathrm{BF_l}$ and can be calculated as:

$$Z^{\mathrm{up}}_{kl} = \sum_{i \in \mathrm{BF(k)}} \sum_{j \in \mathrm{BF(l)}} I^{\mathrm{bf}}_{ik} Z_{ij} I^{\mathrm{bf}}_{jl}$$

with I^{bf}_{ik} the amplitude of the lowerlevel rooftop function over the side i in the upperlevel basis function $\mathrm{BF_k}$.

The quantities we are finally interested in, are the overall current densities I^e (Fig. 1e) due to the excitation of exterior port $\mathrm{P_e}$. These current densities can be found as the linear combination of the profile of each "generalized rooftop" basis function $\mathrm{BF_k}$ (I^{bf}_{ik}), multiplied by its amplitude A_{ke} calculated during the upperlevel MOM:

$$I_{ie} = \sum_{k \in LP} I^{\mathrm{bf}}_{ik} A_{ke}$$

III. ITERATIVE REFINEMENT

During the simulation of the lower level in the Multilevel Moments Method, the presence of metallizations other than the subcircuit considered is neglected. This can be a cause of inaccuracy and errors in the final result. In order to reduce the errors an iterative technique converging to the exact solution, can be put forward.

This work was supported by the Belgian National Fund for Scientific Research (N.F.W.O.).

In our approach we iteratively refine the upperlevel basis functions $\mathrm{BF_k}$ to a set of diakoptic basis functions. The k-th diakoptic basis function can be found by exciting the k-th lowerlevel port $\mathrm{LP_k}$ while leaving the other lowerlevel ports $\mathrm{LP_l}$ open (unexcited). It can be proven that the overall solution based on these basis functions will match the full solution —i.e. the solution from one direct MOM— exactly (apart from numerical inaccuracies). Therefore we iteratively construct the profiles resulting from exciting each of the lowerlevel ports separately. Thus the original subsectional upperlevel "rooftop" basis functions are extended into full-domain diakoptic basis functions.

Our iterative method is based on Modified Diakoptics (MD) [5] and has a physical interpretation (Fig. 2). During the lowerlevel MOM simulation no metallization other than that of the subcircuits connected to the considered artificial port is taken into account (Fig. 2a). The currents on each of these subcircuits will excite currents on all the other subcircuits by (1st order) field coupling (Fig. 2b). During the first iteration these 1st order coupling currents are calculated. These currents will, in turn, excite (2nd order) coupling currents on all the other subcircuits (Fig. 2c). These are calculated in the second iteration. These currents will again excite currents on all the other subcircuits and so forth. The actual current is the sum of the lowerlevel MOM current and the currents from the different iterations (Fig. 2d).

In order to simplify the notation we combine the sides in the complete circuit into blocks of sides: all the sides belonging to the same subcircuit s (excluding the portsides of the connected lowerlevel ports) are combined into one block for each subcircuit (I^{S}_{sk}); all the portsides belonging to the same lowerlevel port $\mathrm{LP_l}$ are also combined into one block for each lowerlevel port (I^{P}_{lk}). The coupling between these blocks of sides are combined into the Z-matrices Z^{S}_{st}, Z^{SP}_{sl} and Z^{P}_{lm}, describing the coupling between the sides of subcircuits (s) and (t) and of lowerlevel ports $\mathrm{LP_l}$ and $\mathrm{LP_m}$.
Using the iterative correction $I^{\mathrm{S}}_{sk(n)} = I^{\mathrm{S}}_{sk(n-1)} + \Delta I^{\mathrm{S}}_{sk(n)}$ and omitting the coupling between the highest order coupling currents, the block iteration formulas for basis function $\mathrm{BF_k}$ become:

$$\Delta I^{\mathrm{S}}_{sk(1)} = \begin{cases} 0 & \text{if } \mathrm{s} = \mathrm{t}_{1,2} \\ -(Z^{\mathrm{S}}_{ss})^{-1}\left[Z^{\mathrm{SP}}_{sk} I^{\mathrm{P}}_{kk} + Z^{\mathrm{S}}_{st_1} I^{\mathrm{S}}_{t_1 k} + Z^{\mathrm{S}}_{st_2} I^{\mathrm{S}}_{t_2 k}\right] & \text{if } \mathrm{s} \neq \mathrm{t}_{1,2} \end{cases}$$

$$\Delta I^{\mathrm{S}}_{sk(n)} = -(Z^{\mathrm{S}}_{ss})^{-1}\left[\sum_{\substack{t=1\\ t\neq k}}^{S} Z^{\mathrm{S}}_{st} \Delta I^{\mathrm{S}}_{tk(n-1)}\right]$$

with $t_{1,2}$ the subcircuits connected to lowerlevel port $\mathrm{LP_k}$.

In the Modified Diakoptics technique, the results of each step in the iteration are updated into the overall currents but after the complete iteration, just as in the standard Block Jacobi linear iterative technique. The convergence can be accelerated by updating the results of each step immediately, resulting in Block Gauss-Seidel (BGS) [6]. The iteration formula then becomes:

$$I^{\mathrm{S}}_{sk(n)} = -(Z^{\mathrm{S}}_{ss})^{-1} \cdot \left[Z^{\mathrm{SP}}_{sk} I^{\mathrm{P}}_{kk} + \sum_{t=1}^{s-1} Z^{\mathrm{S}}_{st} I^{\mathrm{S}}_{tk(n)} + \sum_{t=s+1}^{S} Z^{\mathrm{S}}_{st} I^{\mathrm{S}}_{tk(n-1)}\right]$$

IV. Numerical example.

We applied our technique to the simulation of a lowpass filter (Fig. 3) consisting of an input section with a folded matching stub, a step in width, a central section with three spiral inductors and two patch capacitors, another step in width and finally an output section with a folded matching stub.
This circuit was gridded without and with an edge mesh yielding resp. 349 and 825 unknown variables, divided into 9, 12 and 19 (Fig. 3) parts, and simulated from 1 GHz to 4 GHz. The reference situation was obtained by simulating the complete circuit as a whole ("*full*").

The convergence of the S_{11}-parameter over the complete frequency range for BGS ($n = 349$) is illustrated in Fig. 4. Similar results are obtained for S_{12} (S_{21}) and S_{22}.

Our iterative technique reduces the solution time to 65% for $n = 349$ and 25% for $n = 825$ for an accuracy of -80 dB in the S-parameters. The solution time will decrease more substantially as the number of unknowns further increases.

We tested our approach to other examples such as transmission lines, double stub filters and bandpass filters with very similar results as for the lowpass filter.

References

[1] R.F. Harrington, *Field Computations by Moment Methods*, Macmillan, New York, 1968.

[2] Jeannick Sercu, Niels Faché, Frank Libbrecht, and Paul Lagasse, "Mixed potential integral equation technique for hybrid microstrip-slotline multilayered circuits using a mixed rectangular-triangular mesh," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-43, no. 5, pp. 1162–1172, May 1995.

[3] Khahil Kalbasi and Kenneth R. Demarest, "A multilevel formulation of the method of moments," *IEEE Trans. Antennas Propagat.*, vol. 41, no. 5, pp. 589–599, May 1993.

[4] Chalmers M. Butler, "Diakoptic theory and the moment method," in *IEEE AP-S Int. Symp. Digest*, Dallas, USA, May 8–10 1990, IEEE, vol. I, pp. 72–75.

[5] Felix Schwering, Narinda Nath Puri, and Chalmers M. Butler, "Modified diakoptic theory of antennas," *IEEE Trans. Antennas Propagat.*, vol. 34, no. 11, pp. 1273–1281, Nov. 1986.

[6] R. Van Norton, "The solution of linear equations by the Gauss-Seidel method," in *Mathematical Methods for Digital Computers*, Ralson and Wilf, Eds., chapter 3, pp. 56–61. John Wiley Sons Inc., 10th edition, 1967.

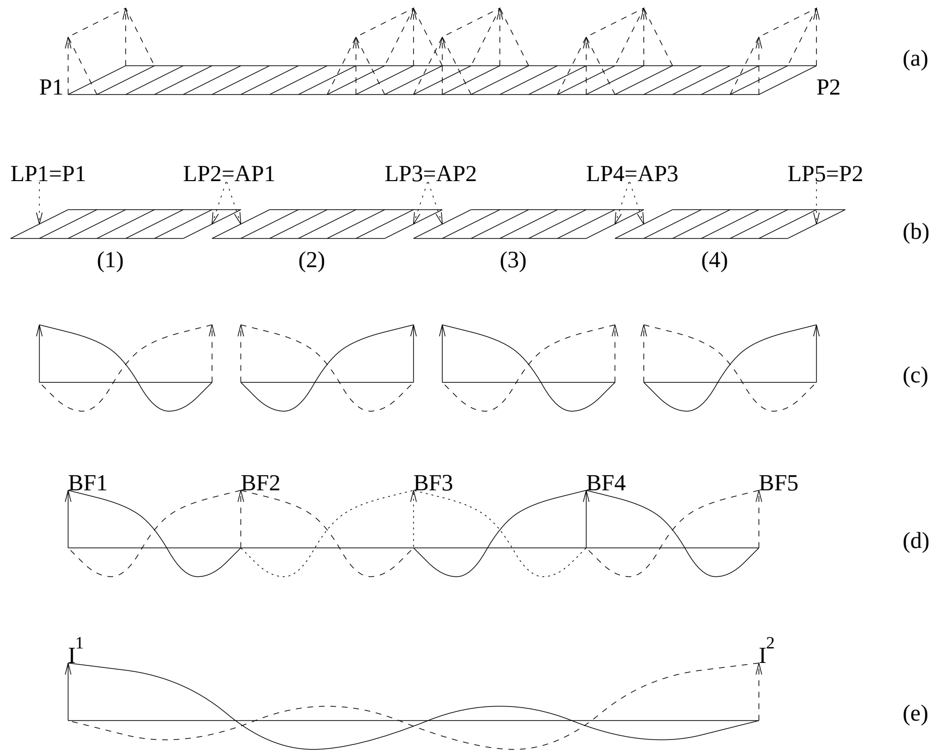

Fig. 1. Two-level Moments Method.

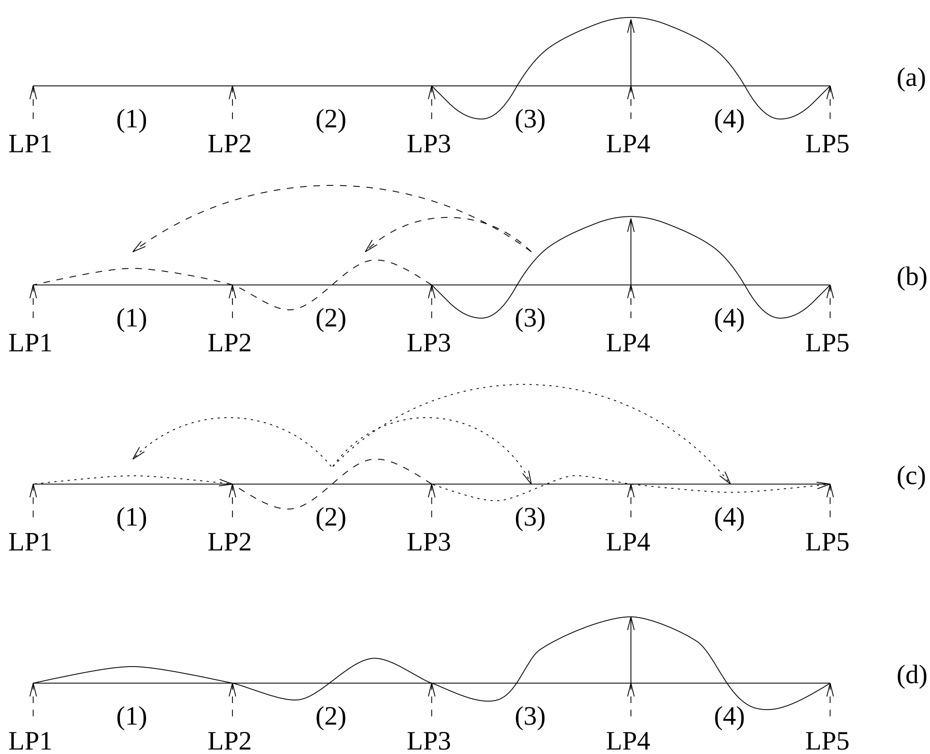

Fig. 2. Modified Diakoptics.

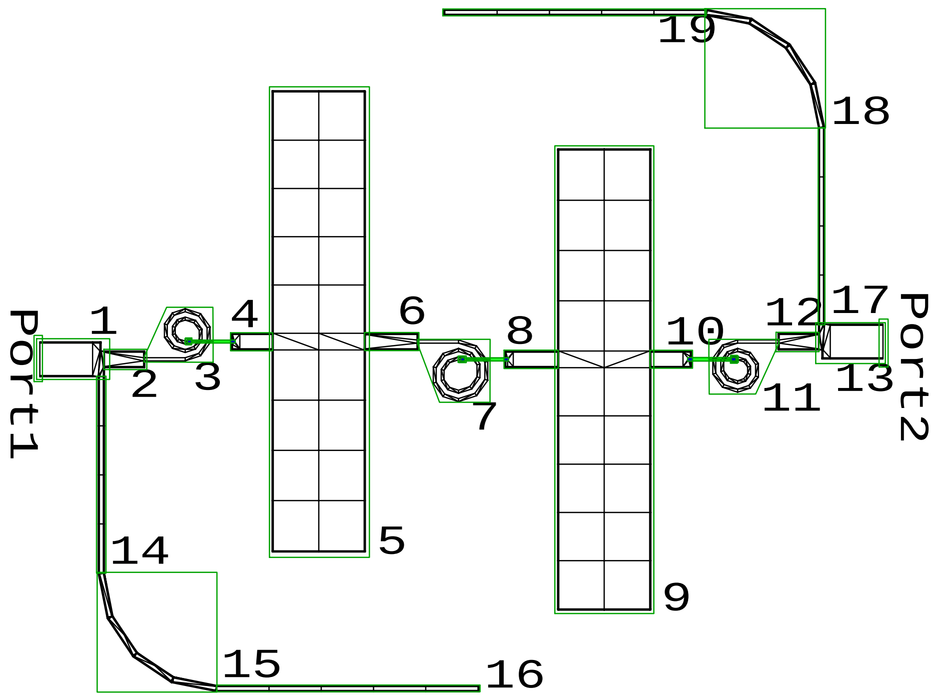

Fig. 3. Lowpass filter with 19 divisions.

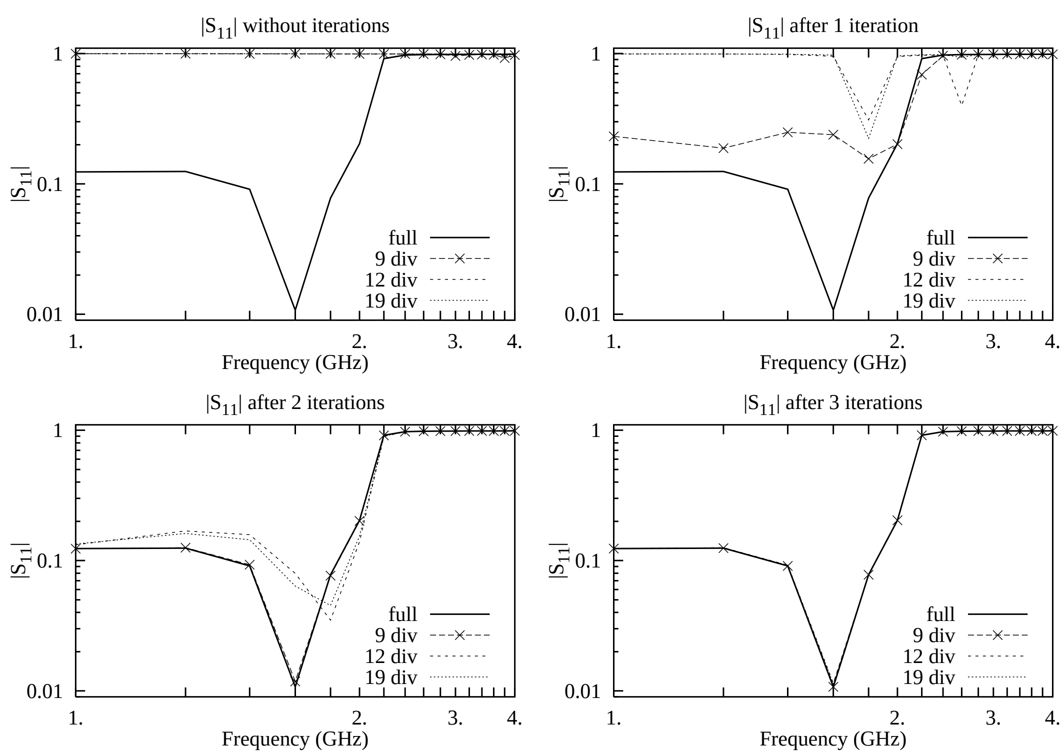

Fig. 4. Convergence of the reflection S-parameter (S_{11}) of the lowpass filter.